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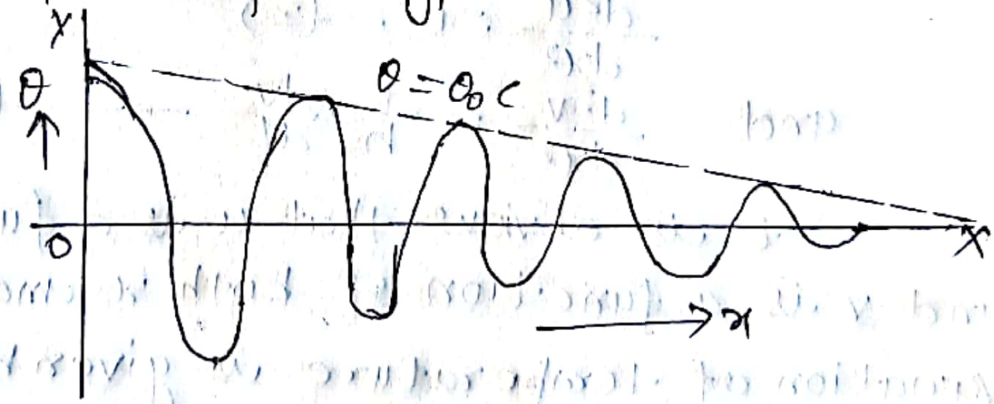
B.Sc(part-1) Hons
paper - II

principal of periodic flow of method of heat.

principal:- If a metallic bar is periodically heated and cooled at one end, the temperature at any point in the bar will also vary periodically. The bar will never reach a steady State but the alternate heating and cooling is continued for a Sufficient number of times. The variation of temperature at any point will attain a fixed character so that the mean temperature at each point remains steady, which may be Called the steady periodic state. Under these Conditions If the periodic variation of temperatures is simple harmonic then the variation of temperature along the bar Can be represented by a Sine curve of the damped type.



Here the amplitude of temprature variation disminishes as the distance from the hot end increases. It Can be analytically represented by

$$\theta = a\sin(\omega t + \beta)$$

where θ is temperature of any point measured from the mean value at the point, t the time, a the amplitude ($=\theta_0 e^{-mx}$) represented by the dotted curve and β is constant.

The fundamental equation of such a periodic flow is given as

$$\frac{d^2\theta}{dx^2} = \frac{\rho c}{K}\cdot\frac{d\theta}{dt} + \frac{EP}{KA}\theta \quad \text{——— (1)}$$

We have to solve this equation such that solution is a periodic function of time. However, when the bar is covered by a guard ring so that it is prevented from losing heat by radiation, then equation (1) reduces to

$$\frac{d^2\theta}{dx^2} = \frac{\rho c}{k}\cdot\frac{d\theta}{dt}$$

or, $$\frac{d\theta}{dt} = \mu\frac{d^2x}{dt^2} \quad \left(\text{where } \mu = \frac{k}{\rho c}\right) \text{ — (2)}$$

solve equation (2) by putting $\theta = u+v$ where u and v satisfy the following relations

$$\frac{d^2u}{dx^2} = 0 \quad \text{— (3)}$$

and $$\frac{d^2v}{dx^2} = \frac{1}{h}\cdot\frac{dv}{dt} \quad \text{——— (4)}$$

It is obvious that u is a function of x only and v is a function of both x and t. If the variation of temperature is given by a simple harmonic function of time, we can assume the solution as

$$v = v_0 e^{-\alpha x} e^{i(\omega t + \beta x)} \quad \text{——— (5)}$$

To find α and β we seek that v given by (5) satisfy relation (4)

Now
$$\frac{dv}{dt} = j\omega v_0 e^{-\alpha x} e^{j(\omega t+\beta x)} = j\omega v$$

and
$$\frac{dv}{dx} = v_0\left[-\alpha e^{-\alpha x} e^{j(\omega t+\beta x)} + j\beta e^{-\alpha x} e^{j(\omega t+\beta x)}\right]$$
$$= (-\alpha+j\beta) v_0 e^{-\alpha x} e^{j(\omega t+\beta x)} = (-\alpha+j\beta)v$$

Also
$$\frac{d^2v}{dx^2} = (-\alpha+j\beta)^2 v$$

Hence equation (4) gives
$$j\omega v = h(\alpha-j\beta)^2 v = h(\alpha^2-\beta^2-2j\alpha\beta)v$$

Equating real and imaginary parts, we get
$$h(\alpha^2-\beta^2) = 0 \quad \text{i.e.} \quad \alpha^2 = \beta^2$$
$$\text{or,} \quad \alpha = \pm\beta \quad \text{and} \quad \omega = -2h\alpha\beta$$

Since $\alpha\beta = -\frac{\omega}{2h}$ is negative, α and β must have opposite sign. Thus we take $\alpha = -\beta$. A damped progressive wave having wavelength λ and period T can also be represented by
$$v = v_0 e^{-\alpha x} \cdot \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda} + \epsilon\right) \quad \text{——(6)}$$

It may be noted that equation (5) and (6) are equivalent.

Comparing equations (5) and (6), we get
$$\omega = \frac{2\pi}{T} \quad \text{and} \quad \beta = -\frac{2\pi}{\lambda}$$
$$\therefore \alpha = \frac{2\pi}{\lambda}$$

Also
$$h = -\frac{\omega}{2\alpha\beta} = \frac{2\pi}{T}\cdot\frac{1}{2}\left(\frac{\lambda}{2\pi}\right)^2$$
$$h = \frac{\lambda^2}{4\pi T} \quad \text{——(7)}$$

$$V = \frac{\lambda}{T} = 2\sqrt{\frac{\pi h}{T}}$$

and $\alpha = \frac{2\pi}{\lambda} = \sqrt{\frac{\pi}{hT}}$

The fluctuation of temperature at any point is given by

$$v = v_0 e^{-x\sqrt{\pi/hT}} \sin\frac{2\pi}{T}\left(t - \frac{x}{x\sqrt{\pi/hT}} + \epsilon T\right)$$

At any instant, the form of this temperature wave is shown in figure. At a particular point along the bar i.e. for fixed x, θ varies harmonically with time, the period being that of the source. The amplitude of temperature oscillations at any point diminishes exponentially as the distance x of that point along the bar increases.

The solution of equation (3) gives the steady temperature at any point and hence also the mean temprature, we have

$$u = \theta_1 \frac{\theta_1 - \theta_2}{l} x$$

Where θ_1 and θ_2 denotes the mean temperature at the ends of the bar.

Therefore the complete solution is

$$\theta = \theta_1 \frac{\theta_1 - \theta_2}{l} x + \theta_0 e^{-2\pi x/\lambda} \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda} + \epsilon\right)$$

From equation (7) we can evaluate h and hence the conductivity of the bar, if we measure λ experimentally and determine T.